

Mobile Sage-Math for Linear Algebra and its Application

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Abstract

Mathematical tools have long held an important place in classrooms. With the innovation of information and communication technologies, many tools have appeared and been adapted for educational purposes. Sage-Math is a popular mathematical software which was released in 2005. This software has efficient features which utilize the internet and can handle most mathematical problems, including linear algebra, algebra, combinatorics, numerical mathematics and calculus. In this paper, we will introduce this powerful software and discuss how it can be used in classes. We also demonstrate the mobile infrastructure of Sage-Math and its mobile-learning environment.

1 Introduction

Throughout the world, IT is the leading source of innovation in education. The leaders of educational activities want comprehensive and simple methods for using IT in their classes. These demands have

*Corresponding author. This work was supported by BK21 Math Modeling HRD Division through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology Grant No.S-2009-0036000. The third author was supported by Priority Research Centers Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology Grant No.2009-0094070

reformed the educational environment. Most countries which have advanced information technology have been interested in the increased application of IT techniques to education. Korea is no exception to this tendency. The Korean government has been interested in the development of IT since 1990, and Korea has a decent internet infrastructure because of this. Due in part to government investments, Korea is generally considered a high-level country in terms of internet connectivity, and most Koreans have easy access to internet services. In the most recent report of Akamai(<http://www.akamai.com>), which is a leading global internet company, Korea was graded as having excellent high speed internet infrastructure and the best average connection speed(11Mbps) in the world [9]. All classrooms in Korea are connected to the internet. Figure 1 gives the average connection speeds of the leading countries in the world.

Country	Q1 09 Mbps	Q1 09-Q4 08 Change	YoY Change
- Global	1.7	+11%	+29%
1 South Korea	11	-28%	-1.1%
2 Japan	8.0	+14%	+10%
3 Hong Kong	7.6	+9.5%	+29%
4 Sweden	6.9	+23%	+42%
5 Romania	5.8	+3.1%	+51%
6 Switzerland	5.7	+12%	+30%
7 Netherlands	5.4	+10%	+28%
8 Denmark	5.1	+14%	+39%
9 Czech Republic	5.0	+14%	+36%
10 Belgium	4.9	+3.3%	+19%
...			
18 United States	4.2	6.4%	+15%

[Figure 1 : The State of the Internet in 2009]

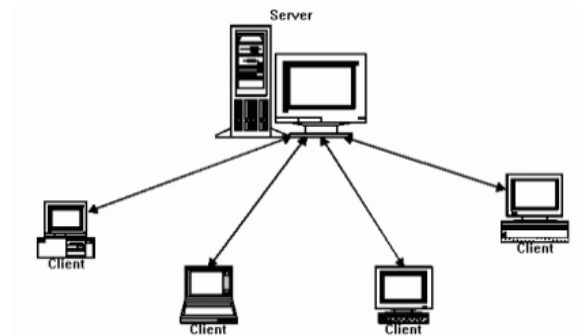
These days, students can easily find good information related to their classes via the internet. The internet is fast and can usually find whatever students want. Instructors should be aware of this and make efforts to incorporate internet resources into classes. To this end, we need to provide more educational content and tools for use by teachers and students(see also [5]).

Unfortunately, to this point most Korean instructors and students have depended on imported tools. As mentioned, Korea has an excellent internet infrastructure, but there are virtually no tools made in the Korean language. Thus, most students must depend on foreign software, such as Mathematica, MATLAB, Maple, GSP, and so forth, in order to solve complex mathematical problems. These imported softwares are fairly expensive in Korea¹. Thus, the desire to use mathematical software leads to grave difficulties. The high cost is one of the main obstacles to using these softwares in Korean schools. Of course, there have been many attempts to find substitutes for these imported softwares, but most of these attempts have not been met with widespread acceptance. The very best tools will be used in the workplace when students get jobs after college, so most students want to use state-of-the-art tools while in college. This situation has not been good for schools, which can generally ill-afford such high costs.

Thus, we need innovative tools which use the familiar structures of expensive commercial softwares such as Mathematica, Maple and MATLAB, but which have moderate prices. It would also be

¹Software prices in Korea(For one copy, 2009): MATLAB: \$1000, Mathematica: \$1399.5, Maple: \$1240

preferable if said software can be used in any location via the internet. Sage-Math is an excellent solution. It has a familiar grammar system, like the commercial softwares, and it follows the GPL(GNU Product License) and is therefore free. Furthermore, Sage-Math has a client-server model(illustrated in Figure 2 below) which is well-adapted to the internet. Therefore, if we use Sage-Math we can see mathematical results in any computer connected to the internet, even if it is a computer with low specifications.



[Figure 2 : Client-Server Model]

Sage-Math is programmed in the Python programming language, which has many functions related to mathematical computation. Since Python has an optimized structure, Sage-Math is fast and accurate. Furthermore, Sage-Math is open source software, so it can be modified freely. Due to these advantages, we can dispense with expensive commercial tools and use Sage-Math in classes without limitation. Sage-Math was developed in 2005 through grants from the NSF(National Science Foundation), and it has been upgraded by many developers after the initial grants program ended. Sage-Math is currently in version 4.1, and will be upgraded by future users(for more information on this, see [4]).

Sage-Math has a client-server model, which makes it even more powerful in the context of the internet. In particular, it can be easily used in countries with good internet connectivity, enabling the entire country to be reached. Expensive commercial software requires an installation by students and instructors, but Sage-Math does not need to be installed by students. If an instructor controls a Sage-Math server, all students in the class need only connect to the instructor's server in order to have access to the program. All computation loads will be borne by the server only, so students have no need of an expensive, high-performance computer. The only requirement for students is to have a computer which can connect to the internet. This is very convenient, both for students and instructors. Any instructor can demonstrate their work to students in the classroom through the internet, without the high-performance computer being present in the classroom. Students can therefore see the visualized results without difficulty, which helps enormously in understanding complicated mathematical concepts.

In particular, Korean students have a strong interest in such a tool. In the survey of computer usage ratio of Korean students, the ratio of assignment performance via the internet is 0.77. This ratio is high compared to the OECD average points²: 0.01. Korean students are now well prepared for internet-based computation tools. As such, Sage-Math should be considered a leading candidate for mathematical requirements.

²M.K. Huh, Analysis of the Korean students, Hankyoreh newspaper, Jan. 25, 2006

To this point in the 21st century, the reform of educational tools and content has been a hot topic. Alan Greenspan, the famous economist who served as the 13th Chairman of the Federal Reserve Bank of the United States, emphasizes the economic importance of improving mathematics and science education in [10].

..... Expanding the number of individuals prepared to use a greater proportion of their intellectual capacity means, among other things, that our elementary and secondary students must broaden their skills in mathematics and related sciences. In my experience, competency in mathematics—both in numerical manipulation and in understanding its conceptual foundations—enhances a person’s ability to handle the more ambiguous and qualitative relationships that dominate our day-to-day decision making. The study of science, of course, also advances problem-solving skills. ...

Korea’s primary surrounding countries, China and Japan, have the same focus on this development (see [1]). If we don’t want to be overtaken in this competition, we need to expend serious effort towards improving our educational system. In undertaking this reform, we need to supply quality tools which most students can easily use before, during, and after their time in college. It was felt that Sage-Math could be an excellent solution for our needs, so a large effort has been made in this area.

2 Sage-Math

2.1 Introduction to Sage-Math

Sage-Math is computational software which can be applied to mathematical problems in linear algebra, combinatorics, numerical analysis, calculus, and so forth. This software is available, free of charge, on the website, <http://www.sagemath.org>. Sage-Math has a client-server model so that a high-specification computer is unnecessary, unlike other commercial mathematical softwares such as Mathematica, MATLAB, Maple, etc. Sage-Math requires nothing more than a computer capable of running a web browser³.

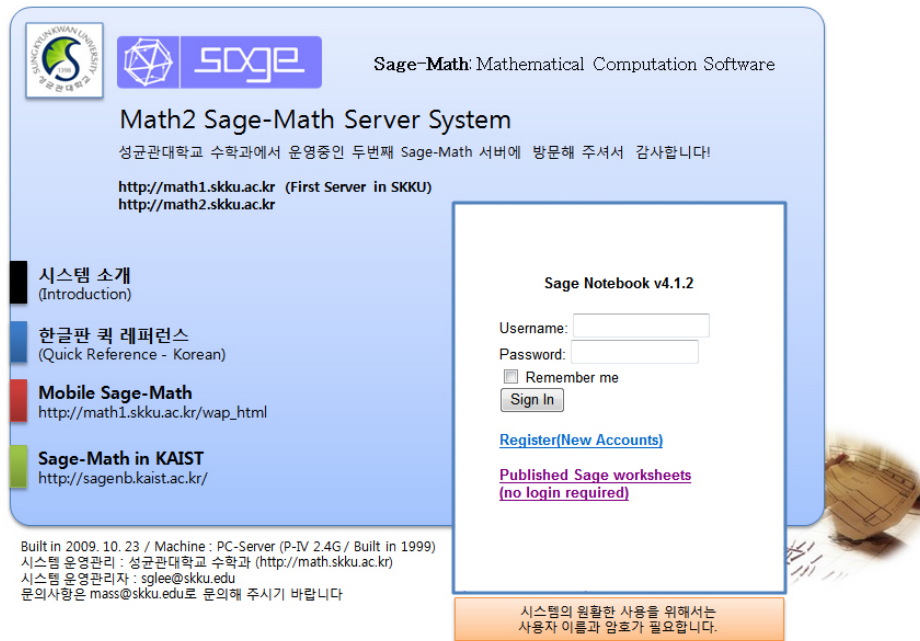
Recently, many other sites have begun carrying this software and we have a demonstrative site which enables and encourages users to further develop Sage-Math and to apply it in the classroom. The primary hosting site is the following:

- Server: <http://math1.skku.ac.kr>
- Username for test-user: test
- Password for test-user: test95

A snapshot of the website is provided in Figure 3. Having navigated to this page, the user can log in using the provided information. To make a new account, it is necessary to connect to an open Sage-Math server⁴. Once logged on to this system, the user can use any functions, which are then evaluated by clicking the “Evaluate” button, or by pressing Shift + Enter on the keyboard.

³Microsoft Internet Explorer, Firefox, Opera etc.

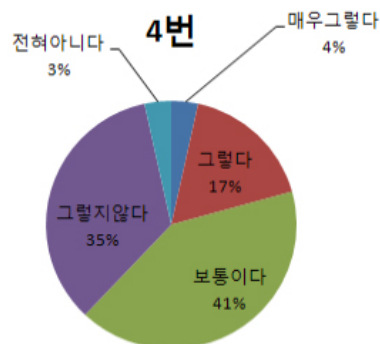
⁴We have an open Sage-Math server at <http://math2.skku.ac.kr>



[Figure 3 : Math2 Sage-Math Server developed in Korean Edition]

2.2 Local development in Korea

Most Korean students have experience with several different mathematical softwares. Most of these softwares are developed in English, however, and the manuals are also written in English. This in an obstacle in getting our students to really appreciate the software. Below, we give the results of the survey in which we asked about the difficulties of classes taught in English. The results indicate that Korean students feel huge difficulties in learning mathematical concepts in languages other than their native tongue. In Figure 4, the purple color indicates that the mathematical classes in English are not favorable to the students and the green color indicates classes in English present greater difficulties in learning mathematical concepts.



[Figure 4 : 2007 BSM Survey Results from Sungkyunkwan University]

It was therefore necessary to develop a Korean edition of the Sage-Math software. Sage-Math has an open license, so the source code is public, and it could be modified to use the Korean language. However, many developers have edited this code. Due to the touch of so many different developers, the source code is complicated, presenting problems in incorporating our local requirements into the software. To illustrate this, we point out that Russian developers intended to make their own version of the software using the Russian language, but were unsuccessful due to the obstacles presented by the complicated source. Nevertheless, we were able to make a Korean edition which includes many modules for treating Korean 2-byte characters. To make this implementation possible, we had to modify the Sage-Math source at the point of the string manager encoding. This is the first edition of Sage-Math done in a language other than English, and now Korean students can use Sage-Math in Korean.

We also have provided a quick reference guide in Korean. Using this reference, shown in Figure 5, students and instructors can find Sage-Math commands easily. This reference is published in the front of our website, /noindent <http://math1.skku.ac.kr>.

<p>Sage 리 레퍼런스: 선형대수학 Robert A. Beezer, 박진영, 이상구 Sage Version 4.1 http://wiki.sagemath.org/quickref GNU Free Document License, extend for your own use Based on work by Peter Jipson, William Stein 한글화: 성균관대학교 수학과</p> <p>벡터 생성 주의: 인덱스는 0부터 시작 $u = \text{vector}(QQ, [1, 3/2, -1])$ 원소가 세 개인 유리수 벡터 $v = \text{vector}(QQ, [2:4, 95:4, 210:0])$ 크기가 210인 유리수 벡터. 3번째와 96번째 원소가 4이다.</p> <p>벡터 연산 $u = \text{vector}(QQ, [1, 3/2, -1])$ $v = \text{vector}(ZZ, [1, 8, -2])$ $2u - 3v$ 일차 결합 $u \cdot \text{dot_product}(v)$ u, v 내적 $u \cdot \text{cross_product}(v)$ u, v 외적 $u \cdot \text{inner_product}(v)$ u, v 내적 $u \cdot \text{pairwise_product}(v)$ 인덱스에 맞춰서 곱 $u \cdot \text{norm}() == u \cdot \text{norm}(2)$ 유클리드 norm $u = \sqrt{\sum u_i^2}$ $u \cdot \text{norm}(1)$ 원소들의 합 $\ u\ _1 = \sum u_i$ $u \cdot \text{norm}(Infinity)$ 최대 원소값 $\ u\ _\infty = \max(u_i)$</p> <p>행렬 생성 주의: 행, 열의 인덱스는 0부터 시작 $A = \text{matrix}(ZZ, [[1,2],[3,4],[5,6]])$ 3 × 2 정수 행렬 $B = \text{matrix}(QQ, 2, [1,2,3,4,5,6])$ 2 × 3 유리수 행렬</p>	<p>$C = \text{matrix}(CDF, 2, 2, [[5+I, 4+I], [1, 6]])$ 53-bit precision을 가지는 2 × 2 복소수 행렬 $Z = \text{matrix}(QQ, 2, 2, 0)$ 영행렬 $D = \text{matrix}(QQ, 2, 2, 8)$ 대각원소가 모두 8인 2 × 2 대각행렬 $I = \text{identity_matrix}(5)$ 5 × 5 단위행렬 $J = \text{jordan_block}(-2,3)$ 3 × 3 행렬, -2 on diagonal, 1's on super-diagonal $\text{var}(x^2 y^2)$; $K = \text{matrix}(SR, [[x,y^2],[0,x^2+2]])$ x, y, z를 미지수로 가지는 2 × 2 symbolic 행렬 $L = \text{matrix}(ZZ, 20, 80, [(5,9):30, (15,77):-6])$ 20 × 80, 5행 9열은 30, 15행 77열은 -6이고 나머지는 모두 0인 정수행렬</p> <p>행렬 곱 $u = \text{vector}(QQ, [1,2,3]), v = \text{vector}(QQ, [1,2])$ $A = \text{matrix}(QQ, [[1,2,3],[4,5,6]])$ $B = \text{matrix}(QQ, [[1,2],[3,4]])$ $u \cdot A, A \cdot v, B \cdot A, B^6, B^(-3)$ 모두 가능 $B \cdot \text{iterates}(v, 6) == v \cdot B^0, v \cdot B^1, \dots, v \cdot B^5$ $\text{rows} = \text{False}$ moves v to right of matrix powers $f(x) = x^2 + 5x + 3$ 이고 $f(B)$ 도 가능 $B \cdot \text{exp}()$ 행렬 exponential, i.e. $\sum_{k=0}^{\infty} \frac{B^k}{k!}$</p> <p>행렬 공간 $M = \text{MatrixSpace}(QQ, 3, 4)$ 차원이 12인 3 × 4 행렬 공간 $A = M([1,2,3,4,5,6,7,8,9,10,11,12])$ M의 원소인 3 × 4 행렬 $M \cdot \text{basis}()$ 기저 $M \cdot \text{dimension}()$ 차원 $M \cdot \text{zero_matrix}()$ M의 원소인 3 × 4 영행렬</p> <p>행렬 연산 $5 \cdot A + 2 \cdot B$ 일차결합</p>	<p>$A \cdot \text{inverse}(), A^(-1), A^{-1}$ A의 역행렬, 만약 A가 singular면 ZeroDivisionError 발생 $A \cdot \text{transpose}()$ 전치행렬 $A \cdot \text{antitranspose}()$ 전치행렬에 역순서 정렬을 위한 행렬 $A \cdot \text{adjoint}()$ 수반행렬 $A \cdot \text{conjugate}()$ 켈레행렬 $A \cdot \text{restrict}(V)$ 벡터공간 V로 제한을 둔 행렬</p> <p>행 연산 Row Operations: (change matrix in place) 주의: 행의 인덱스는 0부터 시작 $A \cdot \text{rescale_row}(1,a) a \cdot (\text{row } 1)$ $A \cdot \text{add_multiple_of_row}(1,j,a) a \cdot (\text{row } j) + \text{row } 1$ $A \cdot \text{swap_rows}(1,j)$ 열 연산도 가능 $\text{row} \rightarrow \text{col}$ 새로운 행렬을 만들자면, $B = A \cdot \text{with_rescaled_row}(1,a)$</p> <p>Echelon Form $A \cdot \text{echelon_form}(), A \cdot \text{echelonize}(), A \cdot \text{hermite_form}()$ 주의: ring에 따라 결과가 다름 $A = \text{matrix}(ZZ, [[4,2,1],[6,3,2]])$ $B = \text{matrix}(QQ, [[4,2,1],[6,3,2]])$ $A \cdot \text{echelon_form}(), B \cdot \text{echelon_form}()$ $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $A \cdot \text{pivots}()$ 열공간에서 대표인자들을 선택 $A \cdot \text{pivot_rows}()$ 행공간에서 대표인자들을 선택</p> <p>부분 행렬 주의: 행, 열의 인덱스는 0부터 시작 $A = \text{matrix}(ZZ, [[4,2,1],[6,3,2]])$ $A \cdot \text{arrows}()$ 행 개수 $A \cdot \text{ncols}()$ 열 개수 $A[1,j]$ 행 1 열 j의 원소</p>
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[Figure 5 : Quick Reference in Korean]

2.3 Implementation in linear algebra classes

Possessing mathematical tools without using them in our classes would be a serious waste of resources. Developed tools must be given an educational implementation, and indeed Sage-Math can be adapted to the classroom. Nowadays we have many mathematical tools, but most can not be efficiently used in the classroom since that is not their intended purpose. Sage-Math is an exception to this. Below, we give a sample application which can be used in a linear algebra class. To make this implementation, we depend on a textbook [3] written in Korean because most students can solve the

textbook problems without software. If they see the computer giving the same result as is found in the textbook, they will realize that the software is reliable and will not hesitate to apply it to real-world problems.

To this end, we have prepared some exercises using Sage-Math at the publication site, <http://math1.skku.ac.kr/pub>. Additionally, the site contains worksheets indicating methods of solving textbook problems using Sage-Math.

- Chapter 4 Section 1 Example 17 : <http://math1.skku.ac.kr/home/pub/253/>
- Chapter 4 Section 2 Example 3 : <http://math1.skku.ac.kr/home/pub/254/>
- Chapter 4 Section 4 Example 4 : <http://math1.skku.ac.kr/home/pub/255/>
- Chapter 4 Section 5 Exercise 3 : <http://math1.skku.ac.kr/home/pub/18/>
- Chapter 5 Section 1 Example 1 : <http://math1.skku.ac.kr/home/pub/40/>
- Chapter 5 Section 4 Example 1 : <http://math1.skku.ac.kr/home/pub/29/>
- Chapter 6 Section 2 Exercise 4 : <http://math1.skku.ac.kr/home/pub/38/>
- Chapter 6 Section 3 Example 2 : <http://math1.skku.ac.kr/home/pub/286/>
- Chapter 6 Section Example 3 : <http://math1.skku.ac.kr/home/pub/4/>
- Chapter 7 Section 3 Example 1 : <http://math1.skku.ac.kr/home/pub/147/>
- Chapter 7 Section 5 Example 4 : <http://math1.skku.ac.kr/home/pub/148/>
- Chapter 7 Section 7 Example 6 : <http://math1.skku.ac.kr/home/pub/43/>
- Chapter 7 Section 8 Example 2 : <http://math1.skku.ac.kr/home/pub/149/>
- Chapter 8 Section 1 Example 2 : <http://math1.skku.ac.kr/home/pub/287/>
- Chapter 8 Section 4 Example 7 : <http://math1.skku.ac.kr/home/pub/256/>
- Chapter 9 Section 1 Exercise P5 : <http://math1.skku.ac.kr/home/pub/54/>
- Chapter 9 Section 2 Example 11 : <http://math1.skku.ac.kr/home/pub/75/>
- Chapter 10 Section 2 Example 2 : <http://math1.skku.ac.kr/home/pub/39/>
- Chapter 10 Section 3 CAS Project 1 : <http://math1.skku.ac.kr/home/pub/53/>

2.4 Example

Now we show an example related with function projections. In what follows, $f(x)$ is a continuous function on $[-\pi, \pi]$, J_N is the space of trigonometric polynomials of order $\leq N$, and $p(x)$ is the orthogonal projection of $f(x)$ onto J_N .

$$p(x) = a_0 + \sum_{n=1}^N (a_n \cos nx + b_n \sin nx)$$

$$\text{where } a_0 = \frac{\langle f(x), 1 \rangle}{\langle 1, 1 \rangle} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{\langle f(x), \cos nx \rangle}{\langle \cos nx, \cos nx \rangle} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{\langle f(x), \sin nx \rangle}{\langle \sin nx, \sin nx \rangle} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

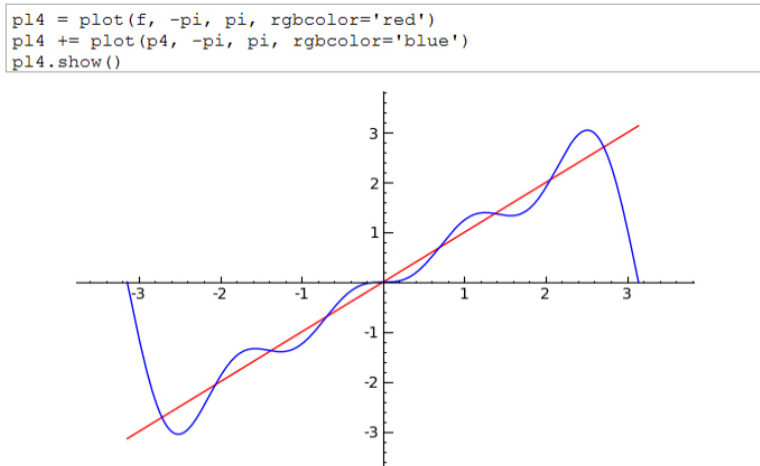
The constants $a_0, a_1, b_1, b_2, \dots$ are called Fourier coefficients. It is in general a complicated problem to find $p(x)$ for an arbitrary $f(x)$ without computational tools, but Sage-Math gives the result by symbolic integration (see [8]) with just a few commands. We can see this process on the website, <http://math1.skku.ac.kr/home/pub/75/>.

```
def Get_JN(f, N):
    a0 = (1/(2*pi)) * integral(f, x, -pi, pi)
    p = a0 + sum([(1/pi) * integral((f * cos(n*x)), x, -pi, pi) * cos(n*x) + (1/pi) * integral((f
    * sin(n*x)), x, -pi, pi) * sin(n*x)) for n in [1 .. N]])
    return(p)
```

The `Get_JN` function finds the trigonometric polynomial of order N for the function f . Let us suppose that $f(x) = x$. We can enter the commands to find $p(x)$ as below.

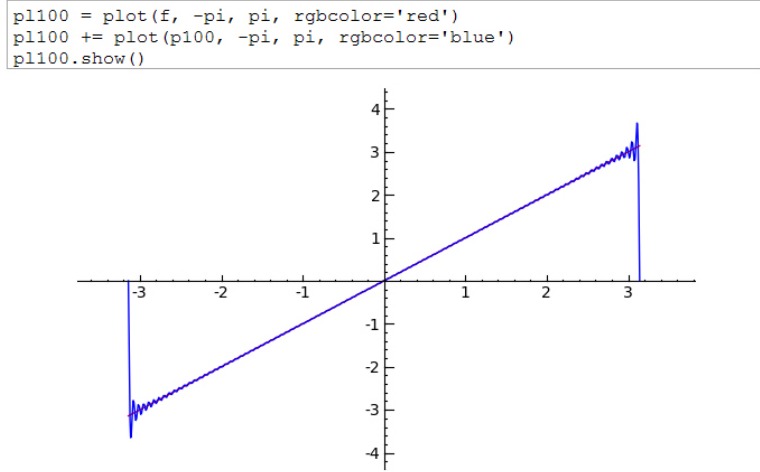
```
f = x;
p4 = Get_JN(f, 4);
```

Using the visualization commands in Sage-Math, we can draw the two functions as in Figure 6 and display the concept of function projection easily.



[Figure 6 : $f(x) = x$ and its projection $p(x)$]

In this way, students can get comprehensive concepts from both the textbook and Sage-Math. The previous example showed only a 4-dimensional space, but we can increase the dimension of the trigonometric projection space. In Figure 7, through some modifications in the previous commands, we show the $N = 100$ case of J_N .



[Figure 7 : $f(x) = x$ and its projection $p(x)$ on J_{100}]

Sage-Math has many powerful functions which help in understanding difficult mathematical concepts. Without the Sage-Math tool, students may meet many difficulties and waste a great deal of time in understanding the concept of function projection. However, Sage-Math helps them understand these concepts without installing new software on their computer. Students only have to connect to the website <http://math1.skku.ac.kr/home/pub/75/>. If they want to modify any parts of other's published works, they can do so very easily⁵. In this way, students can solve and understand hard mathematical problems. This will allow Korean students to tackle complicated, real-world problems wherever they can connect to the internet.

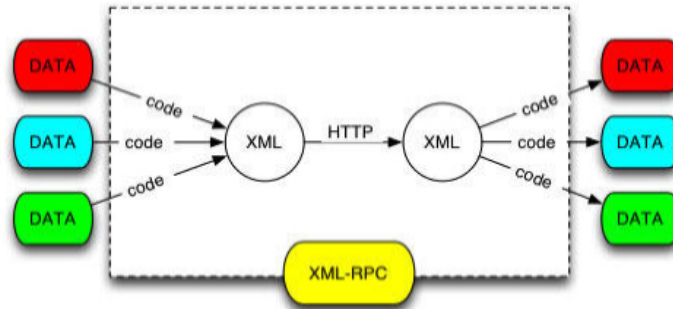
2.5 Mobile module of the Sage-Math

Sage-Math is a network software with special features adapted to the internet. In particular, since it is based on the client-server model, it can evaluate clients' commands on the server. Commonly, this will take place through a computer, but in fact it need not. Sage-Math can be applied on various other platforms which have a connection with the internet, such as MP3 players, cellular phones, etc(see [7]).

This function is based on the XML-RPC technique. The structure of the XML-RPC technique is shown in Figure 8 below. This is a remote procedure call protocol which uses XML to encode its calls and HTTP as a transport mechanism (see [2]). Sage-Math was written in the Python language, and Python supports the XML-RPC functions. Thus, we can use this function to revise Sage-Math source

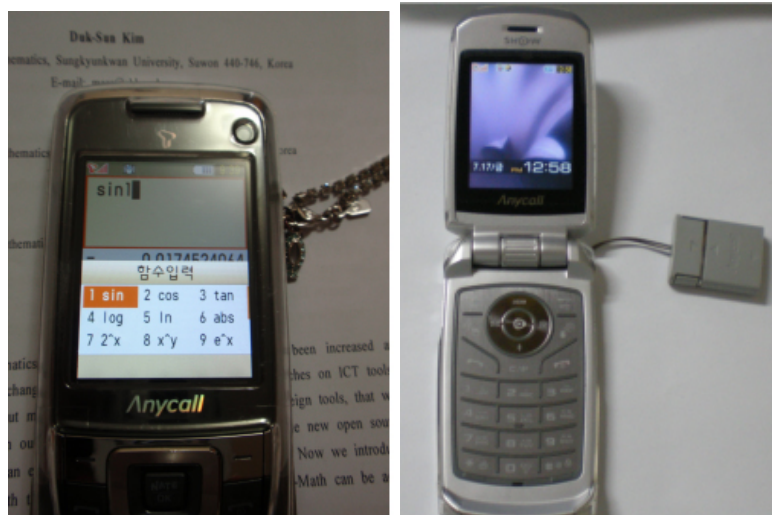
⁵Sage-Math has a convenient function, "Login & Edit this publication sheet".

code. Proceeding in this manner enables all clients which are able to connect to the internet to get the mathematical computation results(including graphic visualizations) from the Sage-Math server. Bloggers can show mathematical results with their own comments added if they can use the XML-RPC codes and the random problem generator (RPG) (see [6]), which is the self training system with randomly generated mathematical problems.



[Figure 8 : Overview of the XML-RPC structure]

XML-RPC can be used on cellular phones, so mathematical results using Sage-Math can be freely displayed in any location. Cellular phones are changed into scientific calculators in this manner. This can also be used in the classroom, which will have a huge effect because most students in Korea use cellular phones such as those shown in Figure 9. Students do not need to spend their time to install specific software on their cellular phones; simple internet connectivity is all which is required. Finally, students are not required to have a high-level cellular phone. The computation load will be entirely borne by the server.

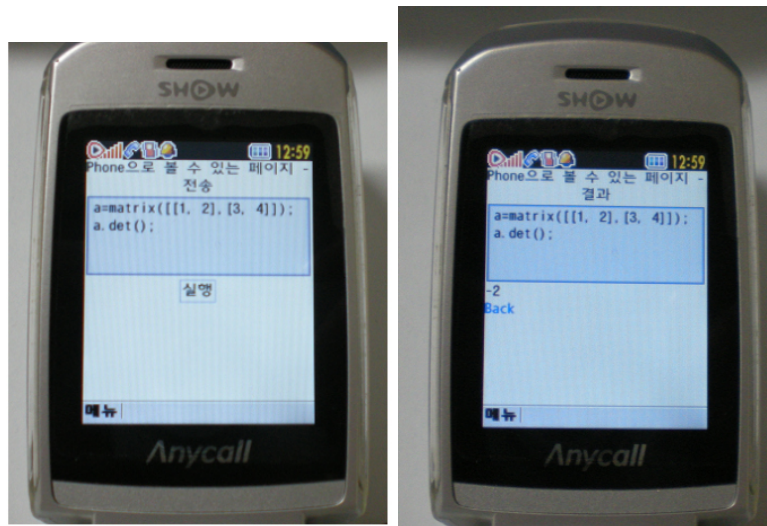


[Figure 9 : Common cellular phones in Korea]

We have made a mobile service module which fits on common cellular phones. To use the mobile service for Sage-Math, connect to the following URL:

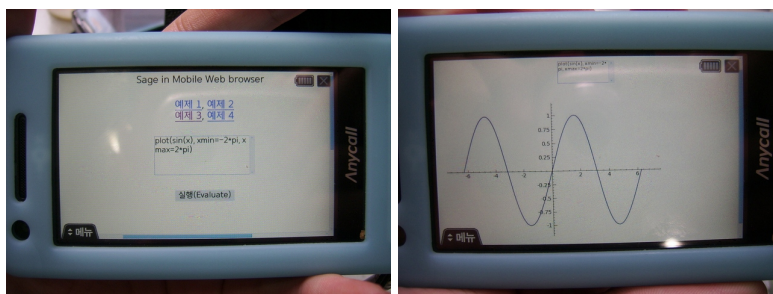
- http://math1.skku.ac.kr/wap_html

When connecting to the mobile service, the user will encounter the screen shown in Figure 10. One can modify commands or input new commands in the windows. After that, click "Execution" to see the results of imputed codes. In order to go back to see earlier results, click the "Back" button. In Figure 10, we show⁶ the determinant of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.



[Figure 10 : Finding a determinant using the cellular phone]

However, entering useful inputs on the cellular phone is difficult. The keypad on the cellular phone is very small and difficult to use compared to the keyboard on a PC. Due to this, we have prepared some examples that are well adapted to the cellphone. The first example shows that we can find the determinant of a matrix. The second example shows that we can factor the given second order polynomial and find its roots. The third example shows the graph of the given function. Finally, the fourth example shows the limits of the given formula and gives the derivatives and integrations. Figure 11 shows⁷ the third example on the cellular phones.



[Figure 11 : Drawing the graphs of $\sin(x)$ using the cellular phone]

⁶The cellular phone in Figure 10 is made by Samsung, Anycall SPH-W5000. The KTF internet service is used.

⁷The cellular phone in Figure 11 is made by Samsung, Anycall W6050. The LG-Oz internet service is used.

To make this module we faced many problems. For example, there were encoding problems, in which all letters were broken. There was also a big problem within the drawing functions. Sage-Math uses only PNG but most cellular phones in Korea didn't support this graphic type. We had to fix all source code to implement those graphic objects, and we gathered all information on cellular phones in Korea. As of 2010, there are three major companies in Korea which support internet connections on cellular phones. The KTF company uses the KUN Web Browser, the SKT company uses the Nate M Browser, and the LGT uses the Lion Browser in their cellular phones. These three web browsers are different, so we had to find a common standard among the three browsers and fix the Sage-Math code to adapt to this common standard. Now we have a mobile module which can be used with any cellular phone in Korea. Furthermore, we have checked that this module can be used on many foreign cellular phones, such as iPhone, iPod, iPad, Nokia (shown in Figure 12), Motorola and so on. Thus, Korean students can use this mobile module without any problems on their cellular phones, both in Korea and abroad.



[Figure 12 : Nokia cellular phone simulator⁸]

2.6 Conclusion

In this research project, we have developed the Sage-Math software for classes in Korea, in particular for linear algebra classes. The Korean educational environment has changed due to improvements in technology, and most students in Korea would like to use the internet more in their studies. Sage-Math is an excellent option for use in our linear algebra classes, and we should obtain additional enhancement from further development of Sage-Math in three key areas: the localized updates in Korean, the implementations of Sage-Math in classes, and the further development of the mobile facility. We have introduced the localization process in Korean, using some examples from linear algebra. Finally, we have introduced our development of the mobile module for use in the ubiquitous

⁸<http://emulator.mtld.mobi/emulator.php>

internet environment. Those processes should aid our students in the process of learning mathematics, and down the road students who have some experience in using those mathematical tools should play an important role in our society.

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